OPERATIONS RESEARCH

SECOND EDITION

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ALWAYS LEARNING

PEARSON

Operations Research

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Preface

Operations research is the study of optimisation techniques. It is applied to decision theory. The existence of optimisation techniques can be traced at least to the days of Newton and Lagrange. Rapid development and invention of new techniques have occurred since the Second World War, essentially because of the necessity to win the war with limited resources. There has been a long-felt need for a simple book on operations research covering the syllabi of Indian universities. The present book has been designed to address this lacuna and cater to the needs of the students of BE, MCA, MBA, M.Sc. and M.Com. courses. It is an outcome of the long-standing experience and interest of the authors in teaching operations research. A unique feature of the book is that each chapter abounds with solved problems that are explained in a simple way that enables the student to understand the subject with ease.

Chapter 1 provides the definition and a brief historical background of operations research. The chapter acquaints the reader with not only the meaning and purpose but also the limitations of operational research. It also gives insight into the approaches and tools of operations research and describes the relationship between an operational research specialist and a manager.

Chapter 2 discusses the formulation of a linear programming problem (LPP) with different types of linear constraints. The graphical solution to an LPP is explained with the aid of a number of examples. The cases of multiple, unbounded solution and infeasible problems are also illustrated graphically. LPPs involving more than two decision variables can be solved with the help of simplex method. Slack variables are introduced to convert less than or equal to type constraints into equations. When some of the constraints are of greater than or equal to type, surplus variables are introduced to convert inequalities to equations. The chapter describes the concept of introducing artificial variables to initiate simple computations. LPPs, in such cases, are solved by two methods, namely the two-phase method and the Big M method. The cases of multiple solution, unbounded solution and infeasible problems are discussed with the help of appropriate examples.

The dual to an LPP is examined in Chapter 3. The economic interpretations of dual variables, which can be used by the management for planning its resources, are also covered in this chapter. If an LPP involving a large number of variables and constraints is to be solved by this method, it will require a large storage space and time on a computer. In this chapter, we take a close look at some computational techniques that have been developed, which require much less computer storage and time than that required by the simplex method. An important and efficient computational technique is the revised simplex method or simplex method with multipliers. The sensitivity analysis of a problem to study the effect of change in various resource levels is also discussed, as are some integer programming formulation techniques. The chapter also highlights the applications of these techniques in managerial problems where LP techniques fail. Two more methods, namely cutting plane method and branch and bound method, as well as the 0-1 programming are elucidated.

In Chapter 4, the formulation of a transportation problem is analysed. Five methods, namely the north-west corner rule, matrix minima method, row minima method, column minima method and Vogel's approximation method, to determine an initial basic feasible solution are spelt out. In addition, the modified distribution method and stepping stone method for obtaining the optimal solution of a

transportation problem are explained. The chapter outlines the unbalanced transportation problem, degenerate transportation problem and trans-shipment problem, while also touching upon sensitive analysis.

Chapter 5 explains the formulation of an assignment problem with the help of examples. The Hungarian method for solving an assignment problem is discussed. The chapter also describes the methods to convert an unbalanced assignment problem into a balanced problem and to handle problems with infeasible assignment. The modification of the assignment problem when the objective is to maximise the objective function is also exemplified.

Chapter 6 is on dynamic programming. It explains the methodology relating to dynamic programming along with various concepts. An assortment of dynamic programming applications is also explained.

The foregoing chapters deal with formulation and solution of models under conditions of perfect information. This is usually referred to as decision-making under certainty. Many managerial decisions, however, are made with some uncertainty. Chapter 7 explores the ways of making decisions under uncertainty, decisions under certainty and decisions under risk. Any decision-making process can be represented by means of a tree known as a decision tree.

Chapter 8 deals with the game theory. The theory of games aids managerial decision-making in a comprehensive environment. After producing a conceptual framework on games, this chapter covers the salient elements of the theory of games, namely saddle mixed strategies, dominance, $2 \times n$ games and $n \times 2$ games.

Chapter 9 elucidates the intricacies associated with the sequencing problem and scrutinizes easier methods of dealing with such problems.

Chapter 10 deals with replacement problems and sheds light on the replacement of items with gradual deterioration, item deterioration with money value and items that failed completely and suddenly. This chapter concludes with the staff replacement problem.

Chapter 11 unravels the various concepts pertaining to inventory. It explains the need, objectives and functions of inventory. Three-way classification of inventory and the factors influencing inventory are also provided. It describes inventory models with probabilistic demand and concludes by discussing Always Better Control (ABC) analysis.

Chapter 12 discusses the basic characteristics of a queuing problem and explains its occurrence in real-life situations. The queuing models under M/M/1, M/M/C and M/Ek/1 systems are illustrated with suitable examples.

The critical path method (CPM) and the project evaluation and review technique (PERT) are discussed in Chapter 13.

Chapter 14 deals with simulation. This technique is used in management problems where it is not possible to use any precise mathematical model. The applications of simulation model in practical business problems are illustrated in this chapter.

The last chapter of this book, Chapter 15, is on non-linear programming (NLP) problems. The necessary and sufficient conditions to obtain optimal solution for an NLP problem given by Kuhn–Tucker are discussed. Further, the concepts of quadratic programming and separable programming are also delineated in this chapter along with their applications.

The Teaching and Learning Package

The teaching and learning package, in the form of PowerPoint lecture slides, can be downloaded from the book's companion Web site www.pearsoned.co.in/amnatarajan. These slides, available for each

chapter, provide lecture outlines, important concepts and diagrams and additional material, which can be used by instructors to deliver effective lectures.

Feedback

Corrections, suggestions and comments for the improvement of the book are appreciated and will be duly acknowledged. To share your feedback on this book, readers can write to pbalu_20032001@yahoo.co.in or angamuthu_tamilarasi@yahoo.co.in.

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Basics of Operations Research

1.1 DEVELOPMENT OF OPERATIONS RESEARCH

Operations research (OR) has its beginning in World War II. The term, operations research, was coined by McClosky and Trefthen in 1940 in the UK. British scientists set up the first field installations of radars during the war and observed the air operations. Their analysis of these led to suggestions that greatly improved and increased the effectiveness of British fighters, and contributed to the success of British defence. Operations research was then extended to anti-submarine warfare and to all phases of military, naval, and air operations, both in Britain and in the United States, and was incorporated in the post-war military establishments of both the countries.

The effectiveness of operations research in military was instrumental in spreading interest in it to other governmental departments and industry. In the USA, the National Research Council formed a committee on operations research in 1951, and the first book on the subject, *Methods of Operations Research* by Morse and Kimball, was published. In 1952, the Operations Research Society of America came into existence. Success of OR in military attracted the attention of industrial managers who were seeking solutions to their complex problems.

Today, almost every large organization or corporation in affluent nations has staff applying operations research, and in government the use of operations research has spread from military to widely varied departments at all levels. This general acceptance to OR has come as the managers have learned the advantage of the scientific approach on which OR is based. Availability of faster and flexible computing facilities and the number of qualified OR professionals has enhanced the acceptance and popularity of the subject. The growth of OR has not been limited to the USA and the UK. It has reached to many countries of the world. Indicative of this is that the International Federation of Operations Research Societies, which was founded in 1959, now comprises member societies from many countries of the world.

India was one of the first few countries who started using OR. In 1949, the first OR unit was established in the Regional Research Laboratory at Hyderabad. At about the same time, another group was set up in the Defence Science Laboratory to solve the problems of stores, purchase and planning. In 1953, an OR unit was established in Indian Statistical Institute, Calcutta, with the aim of using OR method in national planning and survey. The OR Society of India was formed in 1955. The society is one of the first members of the International Federation of OR Societies. The society

started publishing *Opsearch*, a learned journal on the subject in 1963. Today, OR is a popular subject in management institutes and schools of mathematics and is gaining currency in industrial establishments.

Towards the application of OR in India, Prof. Mahalonobis made the first important application. He formulated the Second Five-Year Plan with the help of OR techniques to forecast the trends of demand, availability of resources and for scheduling the complex schemes necessary for developing our country's economy. It was estimated that India could become self-sufficient in food merely by reducing the average food by 15%. Operations research techniques are being used to achieve this goal. Planning Commission made the use of operations research techniques for planning the optimum size of the Caravelle fleet of India Airlines.

In the industrial sector, in spite of the fact that opportunities of OR work at present are very much limited, organised industries in India are gradually becoming conscious of the role of operations research and a good number of them have well trained OR teams. Most popular practical application of OR in India has been mainly of linear programming. With the exception of the government and textile industries, applications of OR in other industries have been more or less equally distributed.

1.2 DEFINITION OF OPERATIONS RESEARCH

Operations research, rather simply defined, is the research of operations. An operation may be called a set of acts required for the achievement of a desired outcome. Such complex interrelated acts can be performed by four types of systems: man, machine man-machine unit and any organization of men, machines, and man-machine units. Operations research is concerned with the operations of the last type of system.

Many definitions of OR have been suggested from time to time. On the other hand, a number of arguments have been put forward as to why it cannot be defined. Perhaps, the subject is too young to be defined in an authoritative way. Some of the different definitions suggested are:

1. Operations research is a scientific method of providing executive departments with a quantitative basic for decisions regarding the operations under their control.

   —Morse and Kimball
2. Operations research in the most general sense, can be characterized as the application of scientific methods, tools and techniques to problems involving the operations of systems so as to provide those in control of the operations with optimum solution to the problem.

   —Churchman, Ackoff, Arnoff
3. Operations research is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive when he tries to achieve a thoroughgoing rationality in dealing with his decision problems.

   —Miller and Starr
4. Operations research is a scientific approach to problem solving for executive management.

   —H M Wagner
5. Operations research is the art of giving bad answers to problems, to which, otherwise, worse answers are given.

   —Thomas L Saaty

6. Operations research is an aid for the executive in making his decisions by providing him with the needed quantitative information based on the scientific methods of analysis.

—C Kittes

7. Operations research is the systematic, method-oriented study of the basic structure, characteristics, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basic for decision-making.

—E L Arnoff and *M J Netzorg*

8. Operations research is the application of scientific methods to problems arising from operations involving integrated systems of men, machines and materials. It normally utilities the knowledge and skill of an interdisciplinary research team to provide the managers of such systems with optimum operating solutions.

—Fabrycky and Torgersen

9. Operations research is an experimental and applied science devoted to observing, understanding and predicting the behavior of purposeful man-machine systems; and operations research workers are actively engaged in applying this knowledge to practical problems in business, government and society.

—Operations Research Society of America

10. Operations research is the application of scientific method by interdisciplinary teams to problems involving the control of organized (man-machine) systems so as to provide solutions which best serve the purpose of the organization as a whole.

—Ackoff and Sasieni

11. Operation research utilities the planned approach (updated scientific method) and an interdisciplinary team in order to represent complex functional relationships as mathematical models for the purpose of providing a quantitative basis for decision-making and uncovering new problems for quantitative analysis.

—Thierauf and Klekamp

12. Operations research is the application of modern methods of mathematical science to complex problems involving management of large systems of men, machines, materials and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system incorporating measurement of factors such as chance and risk to predict and compare the outcomes of alternative decisions, strategies or controls.

—J O R Society, U.K.

1.3 NECESSITY OF OPERATIONS RESEARCH IN INDUSTRY

After having studied as to what is operations research we shall now try to answer as to why to study OR or what is its importance or why its need has been felt by the industry.

As already pointed out, science of OR came into existence in connection with war operations, to decide the strategy by which enemy could be harmed to the maximum possible extend with the help of the available warfare. War situation required reliable decision-making. The need of OR has been equally felt by the industry due to the following reasons:

(a) **Complexity** In a big industry, the number of factors influencing a decision have increased. Situation has become big and complex because these factors interact with each other in a

complicated manner. There is, thus, great uncertainty about the outcome of the interaction of factors like technology, environment, competition, and so on. For instance, consider a factory production schedule which has to take into account:

(i) Customer demand
(ii) Requirements of raw materials
(iii) Equipment capacity and possibility of equipment failure, and
(iv) Restrictions on manufacturing process.

Evidently, it is not easy to prepare a schedule which is both economical and realistic. This needs mathematical models, which in addition to optimization, help to analyse the complex situation. With such models, complex problems can be split up into smaller parts, each part can be analysed separately and then the results can be synthesized to give insights into the problem.

(b) **Scattered responsibility and authority** In a big industry, responsibility and authority of decision-making is scattered throughout the organization and thus the organization, if it is not conscious, *may be following* inconsistent goals. Mathematical quantification of OR overcomes this difficulty also to a great extent.

(c) **Uncertainty** There is a great uncertainly about economic and general environment. With economic growth, uncertainty is also growing. This makes each decision costlier and time consuming. Operations research is thus, quite essential from reliability point of view.

(d) **Knowledge explosion** Knowledge is increasing at a very fast rate. Majority of the industries are not up-to-date with the latest knowledge and are, therefore, at a disadvantage. Operations research teams collect the latest information for analysis purpose which is quite useful for the industries.

1.4 SCOPE/APPLICATIONS OF OPERATIONS RESEARCH

Although the complete list of OR techniques and their applications would fill volumes in itself, the following is an abbreviated set of applications to show how widely these techniques are used today:

1.	Accounting	Cash flow planning
		Credit policy analysis
		Planning of delinquent account strategy
2.	Construction	Allocation of resources to projects
		Determination of proper workforce
		Deployment of workforce
		Project scheduling, monitoring and control
3.	Facilities planning	Factory size and location decision
		Hospital planning
		International logistics system design
		Estimation of number of facilities required

(*Continued*)

		Transportation loading and unloading
		Warehouse location decision
4.	Finance	Dividend policy making
		Investment analysis
		Portfolio analysis
5.	Manufacturing	Inventory control
		Projection marketing balance
		Production scheduling
		Production smoothing
6.	Marketing	Advertising budget allocation
		Product introduction timing
		Selection of product mix
7.	Organizational behavior	Personal justification/planning
		Scheduling of training programmes
		Skills balancing
		Recruitment of employees
8.	Purchasing	Material transfer
		Optimal buying
		Optimal recording
9.	Research and Development	Control of R&D projects
		Product introduction planning.

A similar list can be prepared for any major field of human endeavor. Military activities alone would cover an entire book.

1.5 OPERATIONS RESEARCH AND DECISION-MAKING

Operations research uses the method of science to understand and explain the phenomena of operating systems. It devices the theories (models) to explain these phenomena, uses these theories to describe what takes place under altered conditions, and checks these predictions against new observations. Thus, operations research is a tool employed to increase the effectiveness of managerial decisions as an objective supplement to the subjective feeling of the decision-maker.

For instance, in distribution or allocation areas, OR may suggest the best locations for agencies, warehouses as well as the most economical kind of transportation; in marketing areas, it may aid in indicating the most profitable type, use and size of advertising campaigns in regard to available financial limit. Operations research may suggest alternative courses of action when a problem is analysed

and a solution is attempted. However, the study of complex problems by OR techniques becomes useful only when a choice between two or more courses of action is possible.

Operations research may be regarded as a tool that enables the decision-maker to be objective in creating alternatives and choosing an alternative which is best from among these.

Decision-making is not only the headache of management, rather all of us make decisions. We decide daily about many minor and major issues. The essential characteristics of all decisions are:

1. objective,
2. alternatives at the disposal, and
3. influencing factors.

Once these characteristics are known, one can think of improving the characteristics so as to improve upon the decision itself.

Let us consider a situation where a decision concerns spending summer vacations at a hill resort. The next problem may be to decide the mode of conveyance from amongst the alternatives: train, bus and a taxi.

At the first level of decision-making, bus is chosen as the mode of conveyance just by intuition (may be at random). At the second level of decision-making, the three conveyances are compared and it is decided qualitatively that the bus will be preferred since it is less time consuming than the train and cheaper than a taxi. At the third level of decision-making, the three alternatives are compared and it is suggested that the bus will be chosen, as it will be taking only half the time taken by train and shall be 40% less costlier than the taxi.

Although outcome of all these decisions is the same, one can easily judge the quality of each decision. We may brand the first decision as 'bad' since it is highly emotional, while we may call the second decision as 'good' since it is scientific, though qualitative. The third decision is undoubtedly the best as it is scientific as well as quantitative.

It is this scientific quantification used in OR that helps management to make better decisions.

Advantages of Operations Research Approach in Decision-Making

Following are the salient advantages of an operations research study approach indecision-making:

(i) **Better decisions** Operations research models frequently yield actions that do improve on intuitive decision-making. A situation may be complex so that the human mind can never hope to assimilate all the significant factors without the aid of OR guided computer analysis.

(ii) **Better Coordination** Sometimes operations research has been instrumental in bringing order out of chaos. For instance, an OR oriented planning model becomes a vehicle for coordinating marketing decisions within the limitations imposed on manufacturing capabilities.

(iii) **Better control** The managements of large organizations recognize that it is extremely costly to require continuous executive supervision over routine decision. An OR approach thereby gained new freedom to the executive to devote their attention to more pressing matters. The most frequently adopted application in this category deals with production scheduling and inventory replenishment.

(iv) **Better system** Often, an OR study is initiated to analyse a particular decision problem, such as whether to open a new warehouse. Afterwards the approach is further developed into a system to be employed repeatedly. Thus, the cost of undertaking the first application may produce benefits.

1.6 OPERATIONS RESEARCH IN MODERN MANAGEMENT

The following are some of the roles of operations research in business and management:

1. **Marketing management**
   (a) product selection
   (b) competitive strategies
   (c) advertising strategy
2. **Production management**
   (a) production scheduling
   (b) project scheduling
   (c) allocation of resources
   (d) location of factories and their sizes
   (e) equipment replacement and maintenance
   (f) inventory policy
3. **Finance management**
   (a) cash flow analysis
   (b) capital requirement
   (c) credit policies
   (d) credit risks
4. **Personal management**
   (a) recruitment policies
   (b) assignment of jobs
5. **Purchasing and procurement**
   (a) rules of purchasing
   (b) determining the quality
   (c) determining the time of purchases
6. **Distribution**
   (a) location of warehouses
   (b) size of the warehouses
   (c) rental outlets
   (d) transportation strategies.

1.7 PHASES OF OPERATIONS RESEARCH

Operations research is a logical and systematic approach to provide a rational basis for decision-making. The phase and processes of OR study must also be quite logical and systematic. There are six important steps in OR study, but it is not necessary that in all the studies each and every step is invariably present. These steps are arranged in following logical order.

Step 1 Observe the Problem Environment

Step 1 in the process of OR study is observing the problem environment. The activities that constitute this step are visits, conferences, observations, research and so on. With the help of such activities, the OR scientist gets sufficient information and support to proceed and is better prepared to formulate the problem.

Step 2 Analyse and Define the Problem

Step 2 is analysing and defining the problem. In this step not only the problem is defined, but also uses, objectives and limitations of the study are stressed in the light of the problem. The end result of this step is a clear grasp of need for a solution and understanding its nature.

Step 3 Develop a Model

Step 3 is to construct a model. A model is representation of some real or abstract situation. Operations research models are basically mathematical models representing systems, processes or environment in the form of equations, relationships or formulae. The activities in this step include defining interrelationships among variables, formulating equations, using known OR models or searching suitable alternate models. The proposed model may be field tested and modified in order to work under environmental constraints. The model may also be modified if the management is not satisfied with the answer that it gives.

Step 4 Select an Appropriate Data Input

Garbage in and garbage out is a famous saying. No model will work appropriately if data input is not appropriate. Hence, tapping the right kind of data is a vital step in OR process. Important activities in this step are analyzing internal-external data and facts, collecting opinions using computer data banks. The purpose of this step is to have a sufficient input to operate and test the model.

Step 5 Provide a Solution and Test Reasonableness

Step 5 in OR process is to get a solution with the help of a model and data input. Such a solution is not implemented immediately. First, the solution is used to test the model and to find limitations, if any. If the solution is not reasonable or if the model is not behaving properly, updating and modification of the model is considered at this stage. The end result of this step is a solution that is desirable and supports the current organizational objective.

Step 6 Implement the Solution

Implementation of the solution obtained in previous step is the last step of OR process. In OR the decision-making is scientific and implementation of decision involves so many behavioral issues. Therefore, the implementing authority has to resolve the behavioral issues. He has to sell the idea of use of OR not only to the workers but also to the superiors. Distance between management and OR scientist may offer a lot of resistance. The gap between one who provides a solution and one who wishes to use it should be eliminated. To achieve this, OR scientist as well as management should play a positive role. A properly implemented solution obtained through OR techniques results in improved working and wins the management support.

1.8 MODELS IN OPERATIONS RESEARCH

1.8.1 Classification of Models

The first thing one has to do to use OR techniques after formulating a practical problem is to construct a suitable model to represent the practical problem. A model is a reasonably simplified representation of a real-world situation. It is an abstraction of reality. The models can broadly be classified as

- Iconic (physical) models
- Analogue models
- Mathematical models
- Static models

- Dynamic models
- Deterministic models
- Stochastic models
- Descriptive models
- Prescriptive models
- Predictive models
- Analytic models
- Simulation models.

Iconic model This is a physical, or pictorial representation of various aspect of a system.
Example: Toys, miniature model of a building, scaled up model of a cell in biology, etc.

Analogue or schematic model This uses one set of properties to represent another set of properties which a system under study has.
Example: A network of water pipes to represent the flow of current in an electrical network or graphs, organizational charts and so on.

Mathematical model or symbolic model This uses a set of mathematical symbols (letters, numbers, etc.) to represent the decision variables of a system under consideration. These variables are related by mathematical equations or inequations which describe the properties of the system.
Example: A linear programming model, a system of equations representing an electrical network or differential equations representing dynamic systems, etc.

Static model This a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.
Example: A linear programming problem, an assignment problem, transportation problem.

Dynamic model This model considers time as one of the important variables.
Example: A dynamic programming problem, a replacement problem

Deterministic model Deterministic model is a model which does not take uncertainty into account.
Example: A linear programming problem, an assignment problem etc.

Stochastic model This is a model which considers uncertainly as an important aspect of the problem.
Example: Any stochastic programming problem, stochastic inventory models etc.

Descriptive model Descriptive model is one which just describes a situation or system.
Example: An opinion poll, any survey.

Predictive model This is one which predicts something based on some data.
Example: Predicting election results before actually the counting is completed.

Prescriptive model Prescriptive model is one which prescribes or suggests a course of action for a problem.
Example: Any programming (linear, nonlinear, dynamic, geometric) problem.

Analytic model This is a model in which exact solution is obtained by mathematical methods in closed form.
Example: General linear programming model, specially structured transportation and assignment models.